ABSTRACTS OF PAPERS DEPOSITED AT VINITI*

CALCULATION OF A HEAT PIPE WITH HEAT FLUXES EXCEEDING THE LIMITING VALUE

B. F. Aptekar' and I. M. Blinchevskii

UDC 536.248.2

The problem of the axial temperature profile and the dependence of the thermal resistances of a heat pipe (R_h) and of the source – sink system connected with the heat pipe (R_{s-si}) on the load factor $\varkappa = Q/Q_{lim}$ when the heat flux Q transmitted by the heat pipe exceeds the limiting heat flux Q_{lim} is solved in the one-dimensional approximation. In this case the wick in the heating zone dries partly, since the capillary pump does not ensure the pumping of the liquid heat carrier through the entire length of the heat pipe.

The calculation is carried out in four sections of the heat pipe: a) in the heating zone with a dry wick; b) in the heating zone with a moistened wick; c) in the adiabatic zone; d) in the condensation zone. A number of simplifying assumptions are made and a connection is obtained between $\varkappa$ and the dimensionless profile of the temperature θ_{sh} of the shell of the heat pipe,

$$\varkappa = \int_0^1 \theta_{sh} d\eta^2 \Big/ \int_{\eta_1}^1 \theta_{sh} d\eta^2,$$

where η is the dimensionless axial coordinate; η_1 is the dimensionless coordinate of the end of the dry section.

The dimensionless thermal resistances of the heat pipe and the source – sink system are determined by the equations

$$\rho_h = \frac{(\theta_0 - \theta_4),\ \varkappa > 1}{(\theta_0 - \theta_4),\ \varkappa = 1},\quad \rho_{s-si} = \frac{(\theta_s - \theta_{si}),\ \varkappa > 1}{(\theta_s - \theta_{si}),\ \varkappa = 1},$$

where θ_0, θ_4, θ_s, and θ_{si} are the dimensionless temperatures of the hot and cold ends of the heat pipe and of the heat source and sink, respectively.

The dependence $\theta_{sh}(\eta)$ was calculated by solving the heat-conduction equation for the shell of the heat pipe. The quantity η_1 was obtained from the well-known equation for determining the quantity Q_{lim} in the one-dimensional approximation. The equations obtained are supplemented by the heat-flux balances at the outer surface of the shell of the heat pipe in the heating and condensation zones.

Analytical expressions for the dependences of ρ and $\varkappa$ were obtained for two cases: a) The axial heat conduction of the shell is important; b) it is negligibly small. The temperature profiles along a heat pipe with a partly dry wick and the dependences of ρ and $\varkappa$ were calculated on a computer. It is shown that in the first case the heat pipes and source–sink systems are less sensitive to heat-flux overloads than in the second case. Since in the first case when $Q > Q_{lim}$ the deformation of the temperature profile in the heating zone is not very noticeable, it is desirable to determine the experimental values of Q_{lim} from the bend in the dependence of R_h on Q.

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CALCULATION OF THE SEPARATION RADIUS OF A VAPOR BUBBLE DURING BOILING UNDER VACUUM CONDITIONS

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UDC 536.423.1

An equation proposed earlier for the determination of the separation radius of a vapor bubble during boiling in a vacuum is refined. To determine the values of the coefficient C of the apparent mass in the calculating function

$$R_0 = \left(\frac{\lambda \Delta t}{r\rho''}\right)^{2/3}\left(\frac{49C}{g}\right)^{1/3}$$

we suggest the empirical dependence

$$C = 219 \exp(-3.4P).$$

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PROFILE STABILIZATION OF PLANE FLOW OF A FISSIONABLE MEDIUM IN A QUASI-ONE-DIMENSIONAL APPROXIMATION WITH A BOUNDARY CONDITION OF THE THIRD KIND

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UDC 536.24:532.54

The flow of an ideal Newtonian gas, for which the coefficients of viscosity and thermal conductivity are power-law functions of the temperature T and the internal heat release is $Q \sim 1/T$, in a plane channel of constant cross section is studied theoretically. In contrast to [1], where an analogous problem was solved with a fixed value of T at the channel walls (a boundary condition of the first kind), here the heat flux and T at the channel walls are connected by a linear relation (a boundary condition of the third kind). As in [1], it is assumed that the Mach numbers M are small and the Reynolds and Peclét numbers are large. The steady-state equations of hydrodynamics are decomposed with respect to the small parameter M^2.

The transient process due to the turning on of heat release at the channel entrance, at which a stabilized stream with $Q = 0$ arrives, is studied. On the example of a model problem it is shown that the quasi-one-dimensional description of the two-dimensional situation has an asymptotic character. The profiles of the velocity u and of T are taken from the stabilized solution in implicit form, where T plays the role of the natural transverse coordinate while the values of u and T at the channel axis are the unknown functions. The coefficients of non-one-dimensionality introduced in [1], which appear upon averaging, are generalized to the case of a variable value of T at the wall.

Graphs are presented of the distribution along the length of the channel of the dimensionless values of T and u at the channel axis, as well as of the dimensionless decrease in pressure. As a supplement to [1] it is discovered that the conditional length of stabilization approaches a constant value not only as $Q \to 0$ but also as $Q \to \infty$. When the temperature of the external medium is low enough the ratio of the values of T at the channel wall and axis can vary nonmonotonically along the length of the channel. A boundary condition of the third kind leads to a smaller difference between the values of T at the channel wall and axis than a boundary condition of the first kind.

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Original paper submitted Dec. 31, 1976.

ON THE REACTION OF A FLUID TO A SPHERE OF VARIABLE RADIUS MOVING NEAR A RIGID WALL

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UDC 532.529.6

The problem of the motion of a sphere of variable radius $a(t)$ moving with a velocity u(t) near a plane wall in an ideal fluid is analyzed. The potential of the velocity field, satisfying the Laplace equation and the boundary conditions, can be found by the method of successive mappings.

The kinetic energy of the fluid is found to equal

$$T = \frac{4}{3}\pi\rho a^3\left(\alpha\frac{u^2}{2} - \beta\dot{a}u + \gamma\frac{\dot{a}^2}{2}\right), \tag{1}$$

where the attachment coefficients α, β, and γ have the following form:

$$\begin{aligned}\alpha &= \frac{1}{2}\left[1 + 3\frac{a^3}{c^3} + 3\frac{a^6}{(c^2-a^2)^3} + \cdots\right],\\ \beta &= \frac{3}{2}\frac{a^2}{c^2}\left[1 + \frac{a^3c}{(c^2-a^2)^2} + \frac{a^6}{(c^2-a^2)(c^2-2a^2)^2} + \cdots\right],\\ \gamma &= 3\left[1 + \frac{a}{c} + \left\{\frac{a^2}{c^2-a^2} + \ln\frac{c^2-a^2}{c^2}\right\} + \cdots\right].\end{aligned} \tag{2}$$

The maximum values of these coefficients are reached when the sphere touches the plane and are $\alpha = 0.8038$, $\beta = 0.5326$, and $\gamma = 4.7317$.

When the sphere is moving in an unbounded volume of fluid these quantities are 0.5, 0, and 3, respectively.

It is interesting to note that the term $\beta\dot{a}u$ contained in (1) describes the interaction of the processes of expansion (compression) and translational motion of the sphere.

The reaction of the fluid acting on the sphere is determined from the Lagrange equations corresponding to the generalized coordinates — the position of the center of the sphere and its radius:

$$Q_z = -\frac{d}{dt}\left(\frac{\partial T}{\partial u}\right) + 2\frac{\partial T}{\partial c}, \tag{3}$$

$$Q_a = -\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{a}}\right) + \frac{\partial T}{\partial a}. \tag{4}$$

Equation (3) gives directly the reaction force of the fluid in the direction normal to the wall, while Eq. (4) represents the total force of the pressure acting on the sphere: $Q_a = 4\pi a^2 P$.

When the sphere is touching the plane surface Eqs. (3) and (4) have the form

$$Q_z = -\frac{4}{3}\pi\rho a^2(\alpha-\beta)(3\dot{a}^2 + a\ddot{a}), \tag{5}$$

$$P/\rho = \left(\frac{\gamma-\alpha}{2}\right)\dot{a}^2 + \left(\frac{\gamma-\beta}{3}\right)a\ddot{a}. \tag{6}$$

From (5) it follows that the force acting on an expanding sphere touching the plane is 1.83 times less than the force acting on the sphere in an unbounded volume of fluid with $u = \dot{a}$.

Equation (6) is also of definite interest. This equation can be integrated in the simplest cases. For example, the time of collapse of a spherical cavity touching a rigid surface and having an initial radius a_0 is

$$t_0 \approx 0.949 a_0\sqrt{\frac{\rho}{P_0}}, \tag{7}$$

where P_0 is the pressure in the fluid far from the cavity.

We note that in this case the time of collapse is about 4% greater than the time of collapse of a cavity in an unbounded volume of fluid (the Rayleigh problem).

In the other case, when a spherical cavity grows at a surface under the action of a constant pressure, its growth rate has the form

$$\dot{a} = \sqrt{\frac{2P}{\rho(\gamma-\alpha)}} \approx 0.713\sqrt{\frac{P}{\rho}}. \tag{8}$$

This is approximately 1.2 times less than the growth rate of a spherical cavity in an unbounded fluid.

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EQUATIONS FOR CALCULATING CONTACT THERMAL RESISTANCES AT LOW CLAMPING PRESSURES FOR DETACHABLE JOINTS

E. P. Lippo

UDC 536.21

We find functions for calculating the contact thermal resistances (CTR) at low contact pressures ($0 \le p \le 20 \cdot 10^5$ Pa) for detachable joints at temperatures in the contact zone no higher than 0.3 of the melting pressure of the material in contact. Here the melting temperature is understood to be for that material for which it is lower.

On the basis of the idea of Clausing and Chao concerning the possibility of summing the thermal resistances due to the confinement of the heat-flux streamlines to the macro- and microcontacts, one can write the following equations:

$$R_m = R_{mr} + R_{ms} + R_{mn};\ R_m = R_{mr} + R_{ms};\ R_m = R_{mr} + R_{mn}.$$

Using V. A. Mal'kov's equation for the contact thermal resistance, we obtain the equations for calculating the CTR with allowance only for the nonplane nature of the surfaces in contact.

For sphere-to-sphere contact

$$R_{mn_{sp\text{-}sp}} = -\frac{4}{\pi\sqrt{\pi}} \cdot \frac{\sqrt{S_n}}{\bar{\lambda}_m} \ln\left[\sin\left(\frac{\pi}{2}\sqrt{\frac{S_{c_{sp\text{-}sp}}}{S_n}}\right)\right].$$

For sphere-to-plane contact

$$R_{mn_{sp\text{-}pl}} = -\frac{4}{\pi\sqrt{\pi}} \cdot \frac{\sqrt{S_n}}{\bar{\lambda}_m} \ln\left[\sin\left(\frac{\pi}{2}\sqrt{\frac{S_{c_{sp\text{-}pl}}}{S_n}}\right)\right].$$

For cylinder-to-cylinder contact along the generatrix

$$R_{mn_{c\text{-}c}} = -\frac{2}{\pi} \cdot \frac{S_n}{L\bar{\lambda}_m} \ln\left[\sin\left(\frac{\pi}{2} \cdot \frac{S_{c_{c\text{-}c}}}{S_n}\right)\right].$$

For cylinder-to-plane contact along the generatrix of the cylinder

$$R_{mn_{c\text{-}pl}} = -\frac{2}{\pi} \cdot \frac{S_n}{L\bar{\lambda}_m} \ln\left[\sin\left(\frac{\pi}{2} \cdot \frac{S_{c_{c\text{-}pl}}}{S_n}\right)\right].$$

In writing the equations for the boundary contact areas it is assumed that the deformation is elastic. The radii of curvature of the boundary surfaces in the contact zone entering into these equations are determined from the following equations.

For the cases of sphere-to-sphere and sphere-to-plane contacts

$$R_1 = \frac{1}{2}\left(\frac{S_n}{\pi d_1} - d_1\right), \quad R_2 = \frac{1}{2}\left(\frac{S_n}{\pi d_2} + d_2\right).$$

For the cases of cylinder-to-cylinder or cylinder-to-plane contacts along the generatrix

$$R_1 = \frac{1}{2}\left(\frac{S_n^2}{4L^2 d_1} + d_1\right), \quad R_2 = \frac{1}{2}\left(\frac{S_n^2}{4L^2 d_2} + d_2\right).$$

The well-known empirical equations should be used to determine R_{mr} and R_{ms}.

Equations for the determination of the CTR in cases when the surfaces in contact have macro- and microdeviations of shape, which are usable in practical engineering, are obtained as a result.

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SOLUTION OF SOME HEAT-CONDUCTION PROBLEMS WITH MIXED BOUNDARY CONDITIONS BY THE METHOD OF ELECTRICAL MODELING

G. P. Tarikov

UDC 536.21

We analyze the boundary problem characterized by the differential equation of heat conduction $\Delta T = 0$ and by the boundary conditions $T = V(x, y)$ in the region Σ of the plane $z = 0$, and $q = 0$ outside the region Σ in the plane $z = 0$, where

$$q = -\lambda \operatorname{grad} T.$$

To simplify the problem it is assumed that

$$V(x, y) = V_0 = \text{const}.$$

In the electrical modeling of the heat-conduction problem under consideration we use the analogy which exists between its integral equation and the integral equation characterizing the charge distribution on the surface of a conducting plate having a constant potential.

The modeling is carried out with a quasisteady electric field on a special electrical modeling device.

The error of the experimental results is estimated on the basis of a theoretical and an experimental solution of the problem for a region Σ in the form of an ellipse.

The results of the experimental solution of new problems are presented for the following shapes of the region Σ: a circle with an eccentrically located cutout and a square with a cutout.

The lines of equal values of the function $\psi(x, y)$ for these regions Σ are shown.

Equations for the determination of the function $\psi(x, y)$ for some characteristic cross sections of the regions Σ are obtained as a result of mathematical treatment of the experimental results using the method of least squares and a Minsk-22 computer.

It is concluded that electrical modeling is possible for heat-conduction problems of the type under consideration for regions Σ with arbitrary configurations.

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STABILITY OF A LAMINAR FLAME ON A GRID SURFACE

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UDC 536.45

Heat-resistant metal grids comprise one of the varieties of fire nozzles in infrared gas radiators. The energy source providing for the heating of the grid is the combustion reaction near its surface. With stable combustion the velocity of the gas — air mixture is equal to the normal velocity of the flame. The normal velocity of the flame as a function of the velocity of the gas — air mixture was investigated to clarify the behavior of the combustion zone as the velocity of the gas — air mixture varies. The investigations showed that the distance between the grid surface and the flame is minimal when the ratio of the velocity of the gas — air mixture to the velocity of the adiabatic flame is 0.42. On the basis of the proposed method of determining the position of the flame above the grid calculations were made of the amount of heat imparted to the grid from the flame as a function of the relative velocity for a stoichiometric propane — air mixture. In doing this the temperature of the combustion products was determined from the equations of thermal theory while the amount of heat imparted to the grid was calculated from the change in the heat content of the combustion products. The results of the calculations are in satisfactory agreement with the analogous data of other authors.

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DIRECTIONAL REFLECTIVITY OF A MULTILAYER ABSORBING SYSTEM

I. M. Korzhenevich

UDC 536.3

This paper is devoted to the calculation of multilayer spectrally selective systems having layers of arbitrary thickness. The present calculation is applicable, in particular, to the case of relatively thick layers, (a thickness of 1-10 μm) which have an advantage over experimentally studied thin-layer systems with layers on the order of 10^{-2} μm thick [1, 2] in terms of the technological efficiency of fabrication and the stability.

Earlier calculations of multilayer systems have been made either for nonabsorbing layers or in the case of normal incidence of the radiation [3-5]. But allowance for the absorption of the layers is necessary in principal, since it is precisely to it that the selective properties of the system are due, while the angular dependence of the reflectivity (RE) is very important, as shown in the present paper.

In this paper multilayer systems are calculated with an arbitrary angle of incidence and number of layers with allowance for their absorption. For this purpose the well-known recursion equations (between the cases of N and N — 1 layers) are generalized with allowance for oblique incidence and the absorbing properties of the layers. The quantities entering into these equations — the Fresnel coefficients of reflection at the

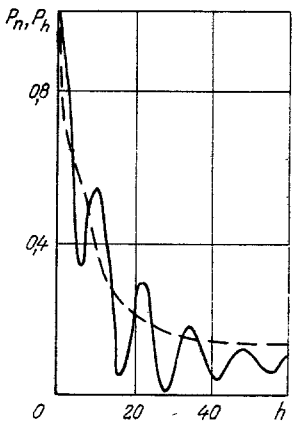

Fig. 1. Dependence of normal (P_n, solid curve) and hemispherical (P_h, dashed curve) reflectivities on film thickness h, μm.

boundaries of the layers, the phase advance, and the extinction within the layers — were calculated as functions of the angle θ of incidence of the radiation on the system and of the index of refraction n and the coefficient of absorption k of the layers.

The result of the calculation is presented in the form of equations for RE, which can serve directly as a computer algorithm.

The RE of several three-layer systems, consisting of SiO and Ge films on metal backings of Au, Ag, Cu, and Al, were calculated as examples. On these examples one sees the strong angular dependence of the RE, leading to a pronounced difference between the hemispherical and normal RE (in previous papers the hemispherical RE was erroneously estimated from the normal RE). The dependence of the normal and hemispherical RE on the thickness $h_{SiO} = h_{Ge} = h$ (μm) of films deposited on a gold backing is presented in Fig. 1 (the wavelength of the radiation is 4 μm).

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Original paper submitted Oct. 8, 1977.

EQUATION OF STATE FOR ETHYL ESTER OF ISOVALERIC ACID

K. D. Guseinov and N. M. Bairamov

UDC 536.71

The density of the ethyl ester of isovaleric acid (with a purity of 99.9%) in the temperature range of 283-600°K and the pressure range of 1-1000 bar was investigated by the method of hydrostatic suspension [1, 2]. The results of the investigations are presented in Table 1. The error of the data is estimated as 0.1%.

Two different equations of state were constructed for the investigated ester in the form

$$P = K(T)\,\rho^2 - L(T)\,\rho^8,$$

$$Pv = RT\,[1 - A(T)\,\rho - B(T)\,\rho^7],$$

where K(T), L(T), A(T), and B(T) are coefficients determined from the test data; $v = 1/\rho$, ρ is the density, g/cm^3; P is the pressure, bar; $R = R_0/\mu$, and $R_0 = 8.3143$ J/mole · deg; μ is the molecular mass.

Using these equations one can calculate the density of the ethyl ester of isovaleric acid up to a temperature of 580°K with an error of 0.05-0.2%; these equations are also suitable for calculating the thermal and some thermodynamic properties.

TABLE 1. Density of the Ethyl Ester of Isovaleric Acid, ρ (kg/m^3)

T, °K	P, bar						
	1	50	200	400	600	800	1000
280	880,2	884,6	895,8	908,9	920,6	931,0	941,0
320	841,8	846,4	860,9	877,2	891,4	903,0	914,0
360	800,8	807,8	825,7	845,8	862,3	876,5	888,0
400	755,5	767,4	789,8	814,3	833,4	850,0	862,9
440	—	722,8	753,5	782,7	805,3	823,6	838,9
480	—	673,4	715,4	751,2	777,0	797,5	815,5
520	—	617,1	676,2	719,7	749,7	773,0	792,2
560	—	546,2	636,2	688,7	723,6	749,4	769,5
600	—	417,0	592,6	657,0	697,5	725,8	747,2

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MENISCUS RADIUS AND DEPTH OF FILLING IN THE MESH CAPILLARY STRUCTURE OF HEAT PIPE

N. N. Kochurova

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To assure the steady operation of a heat pipe in the assigned mode it is necessary that the total pressure drop along the flow line in the vapor and liquid streams not exceed the difference in capillary pressures corresponding to the geometry of the capillary structure of the wick. Thus, the condition for the steady mode of operation of a heat pipe can be written in the following way [1]:

$$\Delta p_l + \Delta p_V \leqslant 2\sigma \left(\frac{1}{R_e} - \frac{1}{R_c}\right), \tag{1}$$

where Δp_l is the pressure loss of the liquid taking place due to friction in the capillary structure, the retardation of the liquid stream by the vapor stream, and the differences in the heights of rise of the meniscus surfaces in the evaporation and condensation zones; Δp_V, pressure losses in the vapor channel; σ, surface tension of heat carrier; R_e, minimum meniscus radius in the evaporation zone; R_c, maximum meniscus radius in the condensation zone. Suppose that the wick of the heat pipe has a mesh structure (Fig. 1). The diameter of the mesh wire is $2a$ and the gap between wires is d. The liquid menisci with a radius R and the boundary wetting angle θ are indicated in the figure. If the level of liquid filling is such that the menisci are located along the line of centers, then $R = d/2\cos\theta$. But if the level is above or below this line then the radius will vary since the distance x between the points of contact of the liquid with neighboring wires will vary and the points themselves will shift along the perimeter. Thus, finding the meniscus radius R comes down to the geometrical problem of calculating the segments x supplementary to d, lying between the straight line $x = a$ and the circle $x^2 + y^2 = a^2$, as well as to the calculation of the angle β (see Fig. 1). We designate the depth of filling of the system with the heat carrier as h. We stipulate that $h = 0$ if the level of the liquid coincides with the tangent to the first row of wires. At the line of centers the depth of filling is $h = a$. Then one can easily obtain an expression for the meniscus radius:

$$R = \frac{d - 2\left(a - \sqrt{2ah - h^2}\right)}{2\cos\left(\theta - \arcsin\frac{|a - h|}{a}\right)}. \tag{2}$$

The meniscus radius will be minimal on the line of centers ($R_{min} = d/2\cos\theta$) and maximal at $h = 0$ or $h = 2a$ [$R_{max} = (d + 2a)/2\sin\theta$]. Thus, when calculating heat pipes with capillary systems having round elements one must allow for the depth h of filling of the system which, as one can show, has an essential effect primarily on the distribution of vapor pressure along the pipe.

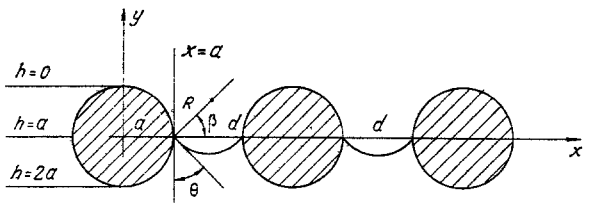

Fig. 1. Geometry of top layer of mesh structure.

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Original paper submitted Nov. 19, 1975.

DETERMINATION OF THERMOPHYSICAL PROPERTIES OF TWO-PHASE SYSTEMS ON THE BASIS OF INTEGRAL CHARACTERISTICS

V. V. Vlasov, N. P. Fedorov, and Yu. S. Shatalov

UDC 536.242:517.3

An analysis is made of the problem of determining the thermophysical properties of materials in a zone of phase transitions when the coefficients of thermal conductivity and diffusivity are piecewise-constant functions of the temperature:

$$\lambda(u) = \begin{cases} \lambda_1, & u < u_p, \\ \lambda_2, & u > u_p, \end{cases} \quad a(u) = \begin{cases} a_1, & u < u_p, \\ a_2, & u > u_p, \end{cases}$$

where u_p is the temperature of the phase transition.

The unknowns are six constants: the coefficients of thermal conductivity λ_1 and λ_2 of the solid and liquid phases, of thermal diffusivity a_1 and a_2 of the solid and liquid phases, and the heat μ and temperature u_p of the phase transition. The coordinate $x = l(t)$ of the phase interface is also unknown.

The experiment is divided into three stages: the heating of the solid material to the melting temperature, the melting process, and the further heating of the melt.

The coefficients of thermal conductivity and diffusivity of the solid (i = 1) and liquid (i = 2) phases are determined from the equations

$$\lambda_i = \frac{q_i^*(p_i)\, L}{\sqrt{(u_i^*(L,\ p_i))^2 - (u_i^*(0,\ p_i))^2}\ \operatorname{Arch} Q_i}, \quad a_i = \frac{p_i L^2}{\operatorname{Arch}^2 Q_i}, \quad Q_i = \frac{u_i^*(L,\ p_i)}{u_i^*(0,\ p_i)},$$

where the integral characteristics of the temperature $u_i^*(x, p_i)$ and of the heat flux $q_i^*(p)$ have the form

$$u_i^*(x,\ p_i) = \int_0^\infty \exp[-p_i t]\, u_i(x,\ t)\, dt, \quad q_i^*(p_i) = \int_0^\infty \exp[-p_i t]\, q_i(t)\, dt.$$

The latent heat μ of the phase transition, the coordinate $x = l(t)$ of the phase interface, and the temperature u_p of the phase transition are determined from the following system of integral equations:

$$\mu = (u_p - h_1)(c_2 - c_1) - \frac{1}{L}\int_0^L [c_2 v_2(x,\ \tau_2) - c_1 v_1(x,\ \tau_1)]\, dx - \frac{1}{\rho L}\int_{\tau_1}^{\tau_2} q(\tau)\, d\tau,$$

$$(c_1 - c_2)\int_0^{l(t)} v_1(x,\ t)\, dx + [(u_p(c_2 - c_1) - \mu]\, l(t) = c_2\int_0^L [v_2(x,\ \tau_2) - v_1(x,\ t)]\, dx + \frac{1}{\rho}\int_t^{\tau_2} q(\tau)\, d\tau,\ u_1(x,\ \infty) = h_1,\ v_i(x,\ \tau) = u_i(x,\ \tau) - h_1,$$

$$u_p = u(0,\ \tau_2),\ u_p = u[l(t),\ t],\ \tau_1 < t < \tau_2,\ u_p = u(L,\ \tau_1).$$

The relative error was estimated on the example of the coefficient of thermal diffusivity from the equation

$$\frac{|\Delta d|}{a} \leqslant 2\,\frac{|\Delta L|}{L} + 2\,\frac{(1 + 1/\bar{Q})\ (\delta + \Delta)}{pu^*(0,\ p)\sqrt{1 - 1/\bar{Q}^2}\ \operatorname{Arch}\bar{Q}},$$

where the parameter p was chosen from the condition of the mimimum of the relative error.

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Original paper submitted Mar. 25, 1977.

NONSTEADY HEAT TRANSFER TO A TWO-DIMENSIONAL REGION WITH VARIABLE PHYSICAL PARAMETERS

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A method published earlier [1] for determining a nonsteady temperature gradient at the boundary of a semiinfinite two-dimensional region $0 \leq x \leq \infty$, $-\infty < y < \infty$ with variable physical parameters from the assigned variation of the temperature $T_0(y, t)$ of the boundary requires the existence of all the derivatives $\partial^n T_0/\partial y^n$, as well as their sufficiently slow growth as a function of the number.

In the case when the parameters of the medium depend only on the x coordinate and the time t it proved possible to so alter the method that it became suitable for functions $T_0(y, t)$ having discontinuities of the first kind along the y coordinate.

The modification of the method is based on the fact that for the "operator root," written earlier in the form

$$\sqrt{\frac{\partial}{\partial t} - \frac{\partial^2}{\partial y^2}}\, T_0(y, t) = \sum_{n=0}^{\infty} (-1)^n \binom{\frac{1}{2}}{n} \frac{\partial^{\frac{1}{2}-n}}{\partial t^{\frac{1}{2}-n}} \frac{\partial^{2n}}{\partial y^{2n}} T_0(y, t),$$

an integrodifferential representation was found:

$$\sqrt{\frac{\partial}{\partial t} - \frac{\partial^2}{\partial y^2}}\, T_0(y, t) = \left(\frac{\partial}{\partial t} - \frac{\partial^2}{\partial y^2}\right) \frac{1}{\sqrt{\pi}} \int_0^t \frac{\left\{\exp\left[(t-\tau)\frac{\partial^2}{\partial y^2}\right]\right\} T_0(y, \tau)}{\sqrt{t-\tau}} d\tau,$$

$$\left\{\exp\left[t_1 \frac{\partial^2}{\partial y^2}\right]\right\} T_0(y, t) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} T_0(y + 2\sqrt{t_1}\, z, t) \exp(-z^2)\, dz.$$

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INVESTIGATION OF THE THERMAL CONDITIONS OF STRUCTURES USING CONJUGATE THERMAL-CONDUCTIVITY EQUATIONS

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One element of the investigation of the thermal conditions of a structure is the determination of the temperature fields in its elements. This problem comes down to the solution of multidimensional thermal-conductivity equations, interrelated in the general case. For constant thermophysical characteristics (λ = const, C_V = const) the solution of such a problem can, according to [1], be represented in the form

$$T(x, t) = \sum_{j=1}^{N} \int_0^t d\tau \int_{D_j} G_j(t, x, \tau, \xi)\, f_j(\tau, \xi)\, d\xi, \tag{1}$$

where $d\xi$ is a surface element of the j-th element (j = 1, 2, ..., N) and N is the number of elements of the thermal model.

When Green's matrix $G(t, x, \tau, \xi)$ is known, this solution is essentially written in explicit form, which is very convenient for conducting thermal design calculations. However, the simplicity of representation of the solution does not compensate for the complexity of the determination of the matrix $G(t, x, \tau, \xi)$.

A solution similar to (1) for the problem of calculating temperature fields in structures can also be obtained when conjugate thermal-conductivity equations are used [2]. In this case, along with the main system of thermal-conductivity equations, one also considers a system of conjugate inhomogeneous equations for which the parameter $P(\mathbf{r}_j, t)$ plays the role of the source function. The investigation of the thermal conditions of a structure is accomplished using the functionals

$$I_\Sigma = \sum_{j=1}^{N} \int_{t_s}^{t_e} \int_{D_j} P(\mathbf{r}_j, t)\, T(\mathbf{r}_j, t)\, dD_j dt = \sum_{j=1}^{N} \int_{t_s}^{t_e} \int_{D_j} q_V(\mathbf{r}_j, t)\, T^*(\mathbf{r}_j, t)\, dD_j dt, \tag{2}$$

where $T^*(\mathbf{r}_j, t)$ is the conjugate temperature. The physical meaning of functional (2) is determined by the parameter of the conjugate equations. When $P(\mathbf{r}_j, t) = \delta(\mathbf{r} - \mathbf{r}_m^0)\,\delta(t - t_e)$, for example, the functional I_Σ has the value

$$I_\Sigma = \sum_{j=1}^{N} \int_{t_s}^{t_e} \int_{D_j} T(\mathbf{r}_j, t)\, \delta(\mathbf{r} - \mathbf{r}_m^0)\, \delta(t - t_e)\, dD_j dt = T(\mathbf{r}_m^0, t_e),$$

i.e., I_Σ is none other than the temperature at the point $\mathbf{r}_m^0$ of the region at the time t_e. We note that, in accordance with (2), the temperature $T(\mathbf{r}_m^0, t_e)$ can also be determined by a heat load different from the nominal one, characterized by the source $q_V(\mathbf{r}_j, t)$ of the thermal-conductivity equations, i.e.,

$$T(\mathbf{r}_m^0, t_e) = I_\Sigma = \sum_{j=1}^{N} \int_{t_s}^{t_e} \int_{D_j} q_V(\mathbf{r}_j, t)\, T^*(\mathbf{r}_j, t)\, dD_j dt.$$

This demonstrates the possibility of determining, using the functionals I_Σ, the quantitative characteristics of the investigated thermophysical process in the range of heat load under consideration without carrying out the repeated solution of the basic system of equations.

Examples of the construction of functionals I_Σ are given in the report.

The approach to the investigation of the thermal conditions of structures which is being discussed is most applicable at the stage of making design thermal calculations, when one must find various parameters characterizing the thermal state of the structure in the presence of various thermal actions.

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